

# Compositional Proof Rules for Multi-Threaded Program Verification (and their automation)

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- Joint work with Grebenshchikov, Gupta, Lopes  
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# “Verification is solving Horn clauses”

- Our view of the program is a transition system

```
int sum (int i) {  
A: int s = 0;  
  
B: while (i > 0) {  
    s = s + i;  
    i = i - 1;  
}  
  
C: assert (s >= 0);  
}
```

$V = (pc, s, i)$

$V' = (pc', s', i')$

$\text{Init}(V) = (pc = A)$

$\text{Step}(V, V') =$

$(pc=A \wedge pc'=B \wedge s'=0 \wedge i'=i) \vee$

$(pc=B \wedge pc'=B \wedge i>0 \wedge s'=s+i \wedge i'=i-1) \vee$

$(pc=B \wedge pc'=C \wedge i\leq 0 \wedge s'=s \wedge i'=i)$

$\text{Error}(V) = (pc=C \wedge s<0)$

# “Verification is solving Horn clauses”

- Find assertion  $inv(v)$  such that

$$init(v) \rightarrow inv(v)$$

$$inv(v) \wedge next(v, v') \rightarrow inv(v')$$

$$inv(v) \wedge error(v) \rightarrow false$$

Transition system is safe

# Safety and termination properties

$\text{Init}(V) \rightarrow \text{Inv}(V)$   
 $\text{Inv}(V) \wedge \text{Step}(V, V') \rightarrow \text{Inv}(V')$   
 $\text{Inv}(V) \wedge \text{Error}(V) \rightarrow \text{false}$

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Transition system is safe

$\text{true} \rightarrow \text{Pre}(n)$   
 $\text{Pre}(n) \wedge n > 0 \rightarrow \text{Pre}(n-1)$   
 $\text{Pre}(n) \wedge n > 0 \wedge \text{Post}(n-1, s) \rightarrow \text{Post}(n, s+n)$   
 $\text{Pre}(n) \wedge n \leq 0 \rightarrow \text{Post}(n, 0)$   
 $\text{Post}(n, s) \rightarrow s \geq 0$

---

Functional program is safe

$\text{Inv}(V) \wedge \text{Step}(V, V') \rightarrow \text{TransInv}(V, V')$   
 $\text{TransInv}(V, V') \wedge \text{Step}(V', V'') \rightarrow$   
 $\quad \text{TransInv}(V, V'')$   
 $\text{dwf}(\text{TransInv}(V, V'))$

$\text{Init}(V) \wedge \text{Step}_i(V, V') \rightarrow T_i(V, V')$   
 $T_i(V, V') \wedge \text{Step}_i(V', V'') \rightarrow T_i(V, V'')$   
 $T_i(V, V') \wedge \text{Step}_i(V', V'') \rightarrow T_i(V', V'')$   
 $(\forall_{j \neq i} \text{Init}(V) \wedge \text{Step}_j(V, V')) \rightarrow E_i(V, V')$   
 $(\forall_{j \neq i} T_j(V, V') \wedge \text{Step}_j(V', V'')) \rightarrow E_i(V', V'')$   
 $\text{Init}(V) \wedge E_i(V, V') \rightarrow T_i(V, V')$   
 $T_i(V, V') \wedge E_i(V', V'') \rightarrow T_i(V, V'')$   
 $T_i(V, V') \wedge E_i(V', V'') \rightarrow T_i(V', V'')$   
 $\text{dwf}(T_1(V, V') \wedge \dots \wedge T_N(V, V'))$

Multi-threaded program terminates

# Safety and termination properties

- Our implementation handles a wide range of verification problems [PLDI'12]
- Verification competitions
  - [TACAS'12] ControlFlowInteger: 91 ok / 93 (2 x time-outs)
  - [TACAS'13] Concurrency: 28 ok / 32 (4 x queue)
- Other tools
  - Blast, CPAchecker, ESBMC, SatAbs, ...

# Proof rules for multi-threaded programs

## **Caveats**

- **Sequential consistency**

## **Pros**

- **not bounded context switches**
- **not only thread-modular proofs**
- **not only data-*race*-free code**

# Owicki-Gries proof rule

- Given a transition system,  
 $N, \text{init}(v), \text{next}(v, v'), \text{error}(v)$
- Find assertions  $R_1(v), \dots, R_N(v)$  such that:
  - $\text{init}(v) \rightarrow R_i(v)$
  - $R_i(v) \wedge \text{next}_i(v, v') \rightarrow R_i(v')$
  - $R_i(v) \wedge (\bigvee_{j \neq i} R_j(v) \wedge \text{next}_j(v, v')) \rightarrow R_i(v')$  } Interference freedom
  - $R_i(v) \wedge \text{error}(v) \rightarrow \text{false}$

Concurrent transition system is safe

# Rely-guarantee proof of safety

- Given a transition system,  
 $N, \text{init}(v), \text{next}(v, v'), \text{error}(v)$

Environment  
assumptions

- Find  $R_1(v), \dots, R_N(v), E_1(v, v'), \dots, E_N(v, v')$ :

$$\text{init}(v) \rightarrow R_i(v)$$

$$(\bigvee_j R_j(v) \wedge \text{next}_j(v, v')) \rightarrow E_i(v, v')$$

}

Environment  
assumptions

$$R_i(v) \wedge (\text{next}_i(v, v') \vee E_i(v, v')) \rightarrow R_i(v')$$

$$R_1(v) \wedge \dots \wedge R_i(v) \rightarrow \text{false}$$

Concurrent transition system is safe



# Rely-guarantee proof of termination

- Given a transition system,  
 $N$ ,  $\text{init}(v)$ ,  $\text{next}(v, v')$ ,  $\text{error}(v)$

- Find assertions  $T_1(v, v')$ , ...,  $T_N(v, v')$ ,  $E_1(v, v')$ , ...,  $E_N(v, v')$ :

$$\text{init}(v) \wedge \text{next}_i(v, v') \rightarrow T_i(v, v')$$

$$T_i(v, v') \wedge \text{next}_i(v', v'') \rightarrow T_i(v, v'')$$

$$T_i(v, v') \wedge \text{next}_i(v', v'') \rightarrow T_i(v', v'')$$

$$(\bigvee_j \text{init}(v) \wedge \text{next}_j(v, v')) \rightarrow E_i(v, v')$$

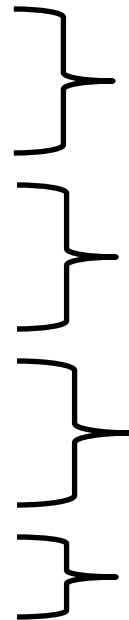
$$(\bigvee_j T_i(v, v') \wedge \text{next}_j(v', v'')) \rightarrow E_i(v', v'')$$

$$\text{init}(v) \wedge E_i(v, v') \rightarrow T_i(v, v')$$

$$T_i(v, v') \wedge E_i(v, v') \rightarrow T_i(v', v'')$$

$$T_i(v, v') \wedge E_i(v', v'') \rightarrow T_i(v, v'')$$

$$\text{dwf}(T_1(v, v') \wedge \dots \wedge T_N(v, v'))$$



Reachable computation  
segments (local)

Environment  
assumptions

Reachable computation  
segments (environment)

Well-foundedness check

**Concurrent transition system terminates**

# Example

```
// Thread 1
lock(l);
while(x>0) {
  x = x-1;
}
unlock(l);
```

```
// Thread 2
while(nondet()) {
  lock(l);
  x = nondet();
  unlock(l);
}
```

- Thread termination for 1?

$$t_1(v, v') = (pc_1 \neq pc_1' \vee \\ l=1 \wedge l'=1 \wedge x'>0 \wedge x'<x)$$

$$E_1(v, v') = (l \neq 1 \vee x'=x)$$

YES

A thread  $i$  is terminating, if no computation contains infinitely many steps of  $i$ .

# Automated verification

## **Caveats**

- **specific theory LI+UIF**

## **Pros**

- **abstraction refinement**
- **bias towards local reasoning**

# Solving recursive Horn clauses over LI+UIF

- Finite unfolding of clauses

- Solve recursion-free clauses

- Generalize solution to unbounded unfolding

# SV-COMP 2013

| Tool                                    | CPAchecker 1.1.10-svcomp13  |      | CSeq  |      | ESBMC 1.2.0      |      | Predator         |       | Threader 0.92    |      |                  |      |
|---|---|------|---|------|------------------|------|------------------|-------|------------------|------|------------------|------|
| Limits                                  | timelimit: 900 s, memlimit: 15360 MB  |      |   |      |                  |      |                  |       |                  |      |                  |      |
| System                                  | CPU: Intel(R) Core(TM) i7-2600K CPU @ 3.40GHz with 4 cores, frequency: 3401 MHz; RAM: 16343684 kB |      |   |      |                  |      |                  |       |                  |      |                  |      |
| Date of run                             | 2012-11-14 06:53  |      | 2012-11-14 08:27  |      | 2012-12-04 12:32 |      | 2012-11-17 03:16 |       | 2012-12-03 03:44 |      | 2012-11-12 05:03 |      |
| Options                                 | -sv-comp13-heap-allocations<br>-heap 320000<br>-disable-java-assertions                           |      | -sv-comp13-heap-allocations<br>-heap 320000<br>-disable-java-assertions |      |                  |      |                  |       | -l java1<br>-r32 |      |                  |      |
| ./sv-comp13-heap-allocations            | status  | time | status  | time | status           | time | status           | time  | status           | time | status           | time |
| pthread/fib_bench_longer_unsafe.cil.c   | unknown   | 1.2  | unknown   | 1.2  | unknown          | 0.35 | unsafe           | 13    | unknown          | 0.04 | unsafe           | 6.0  |
| pthread/fib_bench_unsafe.cil.c          | unknown   | 1.2  | unknown   | 1.2  | unknown          | 0.07 | unsafe           | 3.5   | unknown          | 0.02 | unsafe           | 4.1  |
| pthread/queue_unsafe.cil.c              | unknown   | 1.4  | unknown   | 1.4  | unknown          | 0.07 | timeout          | 900   | unknown          | 0.08 | unknown          | 86   |
| pthread/reorder_5_unsafe.cil.c          | unknown   | 1.4  | unknown   | 1.4  | unknown          | 0.07 | unsafe           | 120   | error            | 0.10 | unsafe           | 2.6  |
| pthread/twostage_3_unsafe.cil.c         | unknown   | 1.5  | unknown   | 1.4  | unknown          | 0.07 | timeout          | 910   | error            | 0.09 | unsafe           | 3.6  |
| pthread/fib_bench_longer_unsafe.i       | unknown   | 1.4  | unknown   | 1.4  | timeout          | 900  | unsafe           | 9.9   | error            | 0.02 | unsafe           | 5.9  |
| pthread/fib_bench_unsafe.i              | unknown   | 1.4  | unknown   | 1.4  | timeout          | 900  | unsafe           | 2.7   | error            | 0.02 | unsafe           | 4.2  |
| pthread/lazy01_unsafe.i                 | unknown   | 1.4  | unknown   | 1.4  | unsafe           | 0.57 | unsafe           | 54    | unknown          | 0.03 | unsafe           | 0.95 |
| pthread/queue_unsafe.i                  | unknown   | 1.7  | unknown   | 1.7  | unknown          | 0.10 | timeout          | 910   | error            | 0.03 | unknown          | 0.41 |
| pthread/reorder_2_unsafe.i              | unknown   | 1.7  | unknown   | 1.8  | unsafe           | 16   | timeout          | 910   | error            | 0.04 | unsafe           | 2.2  |
| pthread/reorder_5_unsafe.i              | unknown   | 1.6  | unknown   | 1.6  | unknown          | 0.16 | timeout          | 910   | error            | 0.03 | unsafe           | 2.5  |
| pthread/stack_unsafe.i                  | unknown   | 1.5  | unknown   | 1.5  | timeout          | 900  | timeout          | 900   | unknown          | 0.02 | unsafe           | 80   |
| pthread/stateful01_unsafe.i             | unknown   | 1.4  | unknown   | 1.4  | unsafe           | 0.68 | unsafe           | 160   | unknown          | 0.02 | unsafe           | 0.91 |
| pthread/twostage_3_unsafe.i             | unknown   | 1.6  | unknown   | 1.7  | unsafe           | 67   | timeout          | 910   | error            | 0.03 | unsafe           | 14   |
| pthread/fib_bench_longer_safe.cil.c     | unknown   | 1.2  | unknown   | 1.2  | unknown          | 0.07 | safe             | 70    | unknown          | 0.03 | safe             | 6.4  |
| pthread/fib_bench_safe.cil.c            | unknown   | 1.2  | unknown   | 1.2  | unknown          | 0.07 | safe             | 18    | unknown          | 0.02 | safe             | 4.6  |
| pthread/queue_ok_safe.cil.c             | unknown   | 1.4  | unknown   | 1.4  | unknown          | 0.07 | unsafe           | 6.7   | unknown          | 0.10 | unknown          | 58   |
| pthread/fib_bench_longer_safe.i         | unknown   | 1.4  | unknown   | 1.3  | timeout          | 900  | safe             | 52    | error            | 0.03 | safe             | 6.4  |
| pthread/fib_bench_safe.i                | unknown   | 1.4  | unknown   | 1.3  | timeout          | 900  | safe             | 13    | error            | 0.02 | safe             | 4.6  |
| pthread/indexer_safe.i                  | unknown   | 1.5  | unknown   | 1.8  | safe             | 0.87 | safe             | 140   | error            | 0.05 | safe             | 4.4  |
| pthread/queue_ok_safe.i                 | unknown   | 1.7  | unknown   | 1.7  | unknown          | 0.09 | timeout          | 900   | error            | 0.03 | unknown          | 0.28 |
| pthread/stack_safe.i                    | unknown   | 1.5  | unknown   | 1.5  | timeout          | 900  | timeout          | 900   | unknown          | 0.04 | safe             | 250  |
| pthread/stateful01_safe.i               | unknown   | 1.4  | unknown   | 1.4  | safe             | 0.78 | safe             | 540   | error            | 0.02 | safe             | 2.7  |
| pthread/sync01_safe.i                   | unknown   | 1.5  | unknown   | 1.5  | unknown          | 0.10 | safe             | 57    | unknown          | 0.03 | safe             | 2.0  |
| pthread-atomic/read_write_lock_unsafe.i | unknown   | 1.4  | unknown   | 1.3  | unsafe           | 1.8  | timeout          | 920   | unknown          | 0.04 | unsafe           | 2.1  |
| pthread-atomic/dekker_safe.i            | unknown   | 1.4  | unknown   | 1.4  | timeout          | 900  | timeout          | 910   | unknown          | 0.02 | safe             | 3.5  |
| pthread-atomic/lamport_safe.i           | unknown   | 1.4  | unknown   | 1.3  | timeout          | 900  | timeout          | 910   | unknown          | 0.02 | safe             | 35   |
| pthread-atomic/peterson_safe.i          | unknown   | 1.4  | unknown   | 1.4  | safe             | 32   | unsafe           | 27    | unknown          | 0.02 | safe             | 4.9  |
| pthread-atomic/read_write_lock_safe.i   | unknown   | 1.4  | unknown   | 1.4  | safe             | 5.4  | timeout          | 920   | unknown          | 0.02 | safe             | 1.6  |
| pthread-atomic/scull_safe.i             | unknown   | 1.5  | unknown   | 1.5  | unknown          | 0.11 | timeout          | 910   | unknown          | 0.03 | safe             | 99   |
| pthread-atomic/szymanski_safe.i         | unknown   | 1.4  | unknown   | 1.4  | safe             | 140  | timeout          | 910   | unknown          | 0.02 | safe             | 13   |
| pthread-atomic/time_var_mutex_safe.i    | unknown   | 1.4  | unknown   | 1.4  | safe             | 1.2  | safe             | 110   | unknown          | 0.02 | safe             | 4.8  |
| total files                             | 32  | 45   | 32  | 46   | 32               | 7500 | 32               | 15000 | 32               | 1.1  | 32               | 720  |
| correct results                         | 0   | 0    | 0   | 0    | 11               | 270  | 15               | 1400  | 0                | 0    | 28               | 570  |
| false negatives                         | 0   | 0    | 0   | 0    | 0                | 0    | 0                | 0     | 0                | 0    | 0                | 0    |
| false positives                         | 0   | 0    | 0   | 0    | 0                | 0    | 2                | 34    | 0                | 0    | 0                | 0    |
| false properties                        | 0   | 0    | 0   | 0    | 0                | 0    | 0                | 0     | 0                | 0    | 0                | 0    |
| score (32 files, max score: 49)         | 0   |      | 0   |      | 17               |      | 15               |       | 0                |      | 43               |      |

- Tools:
  - CSeq
  - ESBMC
  - Threader
- Threader:
  - No reds – no incorrect results wrt. to the assumptions
  - Only 4 unknowns
  - Proofs of correctness and counterexamples
  - Fastest

# SV-COMP 2012

- Tools:
  - ESBMC
  - SatAbs

| Tool                           | CPAchecker ABE r4569   | CPAchecker ABM r4573           | ESBMC 1.17  | FShell 1.3   | Predator         | SatAbs 3.0  |          |         |          |         |          |         |
|--------------------------------|--|--------------------------------|---|--|------------------|---|----------|---------|----------|---------|----------|---------|
| Limits                         | timelimit: 900 s, memlimit: 15000 MB   |                                |   |  |                  |   |          |         |          |         |          |         |
| System                         | os: Linux 2.6.35-30-generic x86_64<br>cpu: Intel(R) Core(TM) i7-2600K CPU @ 3.40GHz<br>cores: 4, frequency: 3401 MHz, ram: 16375440 kB |                                |   |  |                  |   |          |         |          |         |          |         |
| Date of run                    | 2011-12-03 10:32   | 2011-12-04 14:36               | 2011-12-04 08:46  | 2011-12-05 01:14   | 2011-12-04 23:44 | 2011-12-05 13:41  |          |         |          |         |          |         |
| Benchmark                      | concurrency  | concurrency                    | concurrency   | concurrency  | concurrency      | concurrency   |          |         |          |         |          |         |
| Options                        | -heap 12500m<br>-sv-comp12   | -heap 12500m<br>-sv-comp12-abm | --64<br>--error-label ERROR<br>--no-bounds-check<br>--no-div-by-zero-check<br>--no-assertions<br>--no-pointer-check<br>--no-unwinding-assertions<br>--partial-loops<br>--unwind 7 | --unwind 10<br>--query-file benchmarks/fshell_query<br>--no-unwinding-assertions<br>--32 | -m32             | --full-inlining<br>--iterations 500<br>--error-label ERROR<br>--max-threads 5<br>--modelchecker boom<br>--concurrency<br>--32 |          |         |          |         |          |         |
| ../sv-benchmarks/ptthread/     | status   | runtime                        | status  | runtime  | status           | runtime   | status   | runtime | status   | runtime | status   | runtime |
| fib_bench_BUG.cil.c            | unknown  | 1.8                            | unknown   | 1.8  | unsafe           | 14  | error    | 0.14    | unknown  | 0.77    | timeout  | 910     |
| fib_bench_longer_BUG.cil.c     | unknown  | 1.4                            | unknown   | 1.8  | unsafe           | 68  | error    | 0.06    | unknown  | 0.04    | timeout  | 910     |
| queue_BUG.cil.c                | unknown  | 1.5                            | unknown   | 1.7  | unsafe           | 0.18  | error    | 0.07    | unknown  | 0.04    | failure  | 0.22    |
| reorder_5_BUG.cil.c            | unknown  | 1.9                            | unknown   | 1.6  | unsafe           | 38  | error    | 0.07    | unknown  | 0.04    | unsafe   | 1.4     |
| twostage_3_BUG.cil.c           | unknown  | 1.9                            | unknown   | 1.6  | safe             | 900   | error    | 0.08    | unknown  | 0.04    | failure  | 0.11    |
| fib_bench.cil.c                | unknown  | 1.4                            | unknown   | 1.4  | safe             | 25  | error    | 0.06    | unknown  | 0.03    | timeout  | 910     |
| fib_bench_longer.cil.c         | unknown  | 1.8                            | unknown   | 1.4  | safe             | 120   | error    | 0.07    | unknown  | 0.03    | timeout  | 910     |
| queue_ok.cil.c                 | unknown  | 1.5                            | unknown   | 1.9  | safe             | 0.15  | error    | 0.07    | unknown  | 0.04    | failure  | 0.16    |
| total files                    | 8  | 13                             | 8   | 13   | 8                | 1200  | 8        | 0.62    | 8        | 1.0     | 8        | 3600    |
| correct results                | 0  | 0                              | 0   | 0  | 7                | 270   | 0        | 0       | 0        | 0       | 1        | 1.4     |
| false negatives                | 0  | 0                              | 0   | 0  | 1                | 900   | 0        | 0       | 0        | 0       | 0        | 0       |
| false positives                | 0  | 0                              | 0   | 0  | 0                | 0   | 0        | 0       | 0        | 0       | 0        | 0       |
| score (8 files, max score: 11) | <b>0</b>   |                                | <b>0</b>  |  | <b>6</b>         |   | <b>0</b> |         | <b>0</b> |         | <b>1</b> |         |

# Conclusion

- **Compositional** reasoning is useful
- Minimal heap reasoning required  
(some mutexes allocated in the heap)
- Universal and existential arrays properties
- Relevancy of benchmarks

# Extra Slides



# Example: “Fibonacci”

```
#include <pthread.h>
```

```
int i=1, j=1;
```

```
#define NUM 6
```

```
void *t1(void* arg){
```

```
    int k = 0;
```

```
    for (k = 0; k < NUM; k++)    i+=j;
```

```
}
```

```
void *t2(void* arg){
```

```
    int k = 0;
```

```
    for (k = 0; k < NUM; k++)    j+=i;
```

```
}
```

```
int main(int argc, char **argv){
```

```
    pthread_t id1, id2;
```

```
    pthread_create(&id1, NULL, t1, NULL);
```

```
    pthread_create(&id2, NULL, t2, NULL);
```

```
    assert(i <= 377 && j <= 377);
```

```
    return 0;
```

```
}
```

$$R_1(v) = (i \leq 1 \wedge i \leq 2 \wedge i \leq 3 \\ \wedge \dots \wedge i \leq 377)$$

$$R_2(v) = (i \leq 1 \wedge i \leq 2 \wedge i \leq 3 \\ \wedge i \leq 5 \wedge \dots)$$

```

#define SIZE (5)
#define OVERFLOW (-1)
#define UNDERFLOW (-2)

static int top=0, arr[SIZE];
pthread_mutex_t m;

void inc_top(void){ top++;}
void dec_top(void){ top--;}
int get_top(void){ return top;}
int stack_empty(void){ (top==0) ? TRUE : FALSE; }

int push(unsigned int *stack, int x){
    if (top==SIZE) {
        printf("stack overflow\n"); return OVERFLOW;
    } else { stack[get_top()] = x; inc_top(); }
    return 0;
}

int pop(unsigned int *stack){
    if (top==0) {
        printf("stack underflow\n");
        return UNDERFLOW;
    } else { dec_top(); return stack[get_top()];
    }
}

```

```

void *t1(void *arg) {
    int i; unsigned int tmp; for(i=0; i<SIZE; i++)
        pthread_mutex_lock(&m);
    tmp = nondet() % SIZE;
    if ((push(arr,tmp)==OVERFLOW)) assert(0);
    pthread_mutex_unlock(&m);
}

void *t2(void *arg) {
    for(int i=0; i<SIZE; i++) {
        pthread_mutex_lock(&m);
        if (top>0) {
            if (pop(arr)==UNDERFLOW) assert(0);
        }
        pthread_mutex_unlock(&m);
    }
}

int main(void) {
    pthread_t id1, id2;
    pthread_mutex_init(&m, 0);
    pthread_create(&id1, NULL, t1, NULL);
    pthread_create(&id2, NULL, t2, NULL);
    pthread_join(id1, NULL);
    pthread_join(id2, NULL);
    return 0;
}

```

stack\_safe.c

# References

|            |                                     |                           |
|------------|-------------------------------------|---------------------------|
| [POPL'11]  | Solving rec.-free over LI           | with Gupta                |
| [ATVA'10]  | Control abstraction                 | with Gupta                |
| [APLAS'11] | Solving rec.-free over LI+UF        | with Gupta                |
| [TACAS'12] | Termination proof rules             |                           |
| [PLDI'12]  | Solving recursive Horn clauses      | with Grebenschikov, Lopes |
| [CAV'11]   | Threader – tool paper               | with Gupta                |
| [TACAS'12] | HSF – competition contribution      | with Grebenschikov, Lopes |
| [TACAS'13] | Threader – competition contribution |                           |

all papers joint work with Rybalchenko